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# FAILURE PROBABILITY EVALUATION OF A HYDRODYNAMIC JOURNAL BEARING

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**ABSTRACT-** Journal fluid bearings are widely used in industry due to their static and dynamic behavior and their very low coefficient of friction. The technical requirements to improve the new technologies design are increasingly focused on the indicators of dependability of systems and machines. Then, it is necessary to develop a methodology to study the reliability of bearings in order to improve and to evaluate their design quality. Few works are referenced in literature concerning the estimation of the reliability of journal fluid bearings. This paper deals with a methodology to study the failure probability of a hydrodynamic journal bearing. An analytical approach is proposed to calculate static characteristics in using the Reynolds equation. The commonly methods used in structural reliability such as FORM (First Order Reliability Method), SORM (Second Order Reliability Method) and Monte Carlo are developed to estimate the failure probability. The function of performance bounding two domains (domain of safety and domain of failure) is estimated for several geometrical configurations of a hydrodynamic journal bearing (long journal bearings with the hypotheses of Sommerfeld, Gumbel and Reynolds, and a short journal bearing with the hypothesis of Gumbel).

**Keywords:** Hydrodynamic Journal Bearing, FORM, SORM, Monte Carlo, Function of performance, Probability of Failure, Reliability.

## I. INTRODUCTION

Fluid bearings are sensitive components for machines and systems. The design of a fluid bearing is usually based on deterministic static characteristics. However, it is subjected to load and pressure fluctuations or to fluid film gap perturbations induced for instance by defects of the slide ways surfaces geometry [1, 2 and 3]. These factors induce excitations in the bearing dynamic response; which may eventually lead to bearing instability. The prediction of the reliability of a fluid bearing under operating conditions is then necessary for applications requiring high accuracy movements or positioning [1]. Charki and al. [1, 2 and 4] developed a methodology to estimate the failure probability of a thrust fluid bearing and a hemispherical fluid bearing.

The influence of geometrical parameters on the characteristics of bearings such load capacity and stiffness is often studied with a deterministic approach.

Frêne and al. [5] developed an analytical approach with the assumptions of Sommerfeld, Gumbel and Reynolds. Nathi Ram and al. [6] analyzed the behavior of a hybrid journal bearing. An experimental assessment of hydrostatic thrust bearing performance was done by Osman and al. [7].

In reliability analysis, the principle consists in the approximation of a limit-state function bounding two domains (domain of safety and domain of failure). This approach is based on an explicit or implicit performance function evaluated by the solving of the Reynolds equation.

Lemaire and al. [8], detailed the various evaluation methods of the probability of failure. Madsen [9] and Melchers [10] proposed several examples of the estimation of the failure probability. Austin [11] studied the cause of the bearings failures in engines.

This paper presents an adapted method to evaluate the failure probability of a journal fluid bearing (see Fig. 1). The methodology is applied to various geometrical configurations of a hydrodynamic journal bearing.



Fig.1. Failure of circular cylindrical bearing

## II. JOURNAL BEARING MODELING

The expressions of Navier-Stokes equations are considerably simplified. The Reynolds equation [5] is obtained as follows,

Equation of conservation of mass:

$$\vec{\nabla} \cdot \vec{V} = 0 \quad (1)$$

Equation of momentum conservation:

$$\frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \cdot \vec{V} = -\frac{1}{\rho} \nabla P + \frac{\mu}{\rho} \nabla^2 \vec{V} \quad (2)$$

Where  $\vec{V}$  the velocity vector of the fluid with components is  $u, v, w$ ;  $t$  represents the time;  $P$  is the pressure of the fluid.

Boundary conditions [5]:

$$\begin{array}{lll} y = 0 & u = u_1; & v = 0; & w = w_1 \\ y = h & u = u_2; & v = v_2; & w = w_2 \end{array}$$

The equation of Reynolds is expressed as:

$$\begin{aligned} & \frac{\partial}{\partial x} \left[ \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{\mu} \frac{\partial P}{\partial z} \right] \\ &= 6(u_1 - u_2) \frac{\partial h}{\partial x} + 6(w_1 - w_2) \frac{\partial h}{\partial z} + 6h \frac{\partial}{\partial x} (u_1 + u_2) \\ &+ 6h \frac{\partial}{\partial z} (w_1 + w_2) \\ &+ 12v_2 \end{aligned} \quad (3)$$

The hypotheses relative to the equation of Reynolds which allows to write the laminar flow of a fluid between two walls very close and being able to be in movement are given by [5].

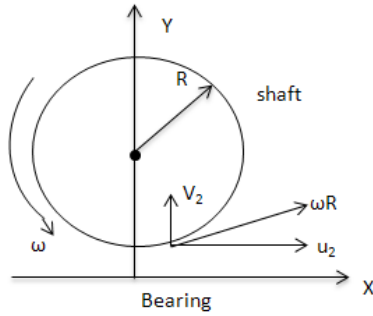


Fig.2. Movement between shaft and bearing

By geometrical considerations (see Fig. 2), we have:

$$\begin{aligned} u_2 &= \omega R \cos \delta \\ v_2 &= \omega R \sin \delta \end{aligned} \quad (4)$$

For an angle  $\delta$  very small, the components of speeds become:

$$u_2 = \omega R \quad (6)$$

$$v_2 = \omega R \frac{\partial h}{\partial x} \quad (7)$$

Replacing components of speed  $u_2$  and  $v_2$  by their expressions, the equation (3) becomes then:

$$\frac{\partial}{\partial x} \left[ \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{h^3}{\mu} \frac{\partial P}{\partial z} \right] = 6\omega R \frac{\partial h}{\partial x} \quad (8)$$

Expression of the film thickness

We consider a point  $M$  along the surface of the bearing and located by the angle  $(\vec{MO}_2, \vec{MO}_1)$  with  $O_1$  et  $O_2$  respectively the centers of the pivot shaft and the bearing (see Fig. 3). The point  $M'$  is the orthogonal projection of  $O_2$  on the line  $(O_1 M)$ .

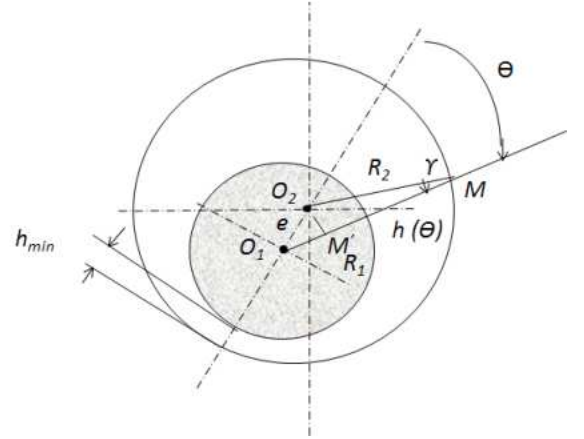


Fig.3. Film thickness between shaft and bearing

Taking into account the relative eccentricity  $\varepsilon = \frac{e}{C}$  varying from 0 to 1, the expression the film thickness  $h(\theta)$  becomes then:

$$h(\theta) = C(1 + \varepsilon \cos \theta) \quad (10)$$

For the calculation, we are going to consider that the load supported by the cylindrical bearing is constant in intensity and in direction.

### III. LONG BEARING WITH SOMMERFELD CONDITIONS

Boundary conditions of Sommerfeld [5] are defined as:

$$\begin{cases} P(\theta = 0, z) = P_0 \\ P(\theta = 2\pi, z) = P_0 \end{cases}$$

Equation Reynolds for a long cylindrical bearing with Sommerfeld conditions becomes:

$$\frac{\partial}{\partial x} \left[ \frac{h^3}{\mu} \frac{\partial P}{\partial x} \right] = 6\omega R \frac{\partial h}{\partial x} \quad (10)$$

With,  $x = R\theta$  replacing  $h(\theta) = C(1 + \varepsilon \cos \theta)$  in the equation and according to an integration of  $2\pi$  we obtain:

$$\begin{aligned} P_S &= \frac{6\mu\omega}{(1 - \varepsilon^2)^{3/2}} \left( \frac{R}{C} \right)^2 \left\{ \psi - \varepsilon \sin \psi \right. \\ &\quad \left. - \frac{\psi(2 + \varepsilon^2) - 4\varepsilon \sin \psi + \varepsilon^2 \sin \psi \cos \psi}{2 + \varepsilon^2} \right\} \\ &+ P_0 \end{aligned} \quad (11)$$

With  $\psi$  such as:

$$\cos\psi = \frac{\cos\theta - \varepsilon}{1 + \varepsilon\cos\theta} \quad (12)$$

The load capacity with Sommerfeld conditions is expressed as:

$$W_S = \frac{12\pi\mu\omega R^3 L \varepsilon}{C^2(2 + \varepsilon^2)(1 - \varepsilon^2)^{1/2}} \quad (13)$$

#### IV. LONG BEARING WITH GUMBEL CONDITIONS

Boundary conditions of Gumbel [5] are defined as:

$$\begin{cases} P(\theta = 0, z) = 0 \\ P(\theta = \pi, z) = 0 \\ P(\theta, z) = 0 \quad \text{si } \pi < \theta < 2\pi \end{cases}$$

$$P_G = \frac{6\mu\omega}{(1 - \varepsilon^2)^{3/2}} \left( \frac{R}{C} \right)^2 \left\{ \psi - \varepsilon \sin\psi - \frac{\psi(2 + \varepsilon^2) - 4\varepsilon \sin\psi + \varepsilon^2 \sin\psi \cos\psi}{(1 - \varepsilon^2)} \right\} + P_0 \quad (14)$$

The integration of the pressure on the surface of the journal bearing gives the following expression of load capacity with Gumbel conditions:

$$W_G = 6\mu\omega L \frac{R^3 \varepsilon (4\varepsilon^2 + \pi^2(1 - \varepsilon^2))^{1/2}}{C^2 (2 + \varepsilon^2)(1 - \varepsilon^2)} \quad (15)$$

#### V. LONG BEARING WITH REYNOLDS CONDITIONS

Boundary conditions of Reynolds [5] are defined as:

$$\begin{cases} P(\theta = 0, z) = P_0 \\ P(\theta_R = 0, z) = 0 \\ \frac{\partial P}{\partial \theta}(\theta = \theta_R, z) = \frac{\partial P}{\partial z}(\theta = \theta_R, z) = 0 \\ P(\theta, z) = 0 \quad \text{si } \theta_R < \theta < 2\pi \end{cases}$$

$$P_R = \frac{6\mu\omega}{(1 - \varepsilon^2)^{3/2}} \left( \frac{R}{C} \right)^2 \left[ \psi - \varepsilon \sin\psi - \frac{\psi(2 + \varepsilon^2) - 4\varepsilon \sin\psi - \varepsilon^2 \sin\psi \cos\psi}{2(1 - \varepsilon \cos\psi_R)} \right] \quad (16)$$

The equation of the fluid break is given:

$$\varepsilon(\sin\psi_R \cos\psi_R - \psi_R) + 2(\sin\psi_R - \psi_R \cos\psi_R) = 0 \quad (17)$$

The load capacity with Reynolds conditions is expressed as:

$$W_R = \frac{3\mu\omega R L}{(1 - \varepsilon \cos\psi_R)(1 - \varepsilon^2)^{1/2}} \left( \frac{R}{C} \right)^2 \left[ \varepsilon^2 \frac{(1 - \cos\psi_R)^4}{1 - \varepsilon^2} + 4(\sin\psi_R - \psi_R \cos\psi_R)^2 \right]^{1/2} \quad (18)$$

#### VI. SHORT JOURNAL BEARING

Two main hypotheses allowing the justification of a short journal bearing are.

- The ratio of the length on the diameter of the bearing is low  $\left( \frac{L}{D} \leq \frac{1}{8} \right)$ .
- The gradient of pressure at the circumference is negligible in front of the axial pressure [5].

Taking into account these hypotheses, the equation of Reynolds for a short journal bearing becomes as follows:

$$\frac{\partial}{\partial z} \left[ \frac{\rho h^3}{\mu} \frac{\partial P}{\partial z} \right] = 6\omega \frac{dh}{d\theta} \quad (19)$$

$$\frac{\partial P}{\partial z} = 0 \quad (20)$$

We obtain finally for short bearing conditions, the expression of the pressure and the load capacity:

$$P(\theta, z) = -\frac{3\mu\omega}{C^2} \left( z^2 - \frac{L^2}{4} \right) \frac{\varepsilon \sin\theta}{(1 + \varepsilon \cos\theta)^2} \quad (21)$$

$$W_C = \frac{\mu R \omega L^3}{4C^2} \frac{\varepsilon}{(1 - \varepsilon^2)^2} (\pi^2(1 - \varepsilon^2) + 16\varepsilon^2)^{1/2} \quad (22)$$

#### VII. PRINCIPLE OF RELIABILITY

- Causes and failure modes of journal bearings

The causes of the failure of a fluid journal bearing are numerous and varied. We distinguish from it essentially: loads excessive (axial and radial), vibrations and shocks, bad alignment, etc.

Failure modes are generally a combination of constraints which act on the bearing until cause a damage or a failure. Failure modes represent the result or the way the problem shows itself and not the cause of the problem of the bearing. We distinguish essentially failure modes due to corrosion or fatigue, or misalignment of the shaft in the bearing [11].

- Function of performance

The reliability of a journal bearing is defined by the knowledge of a function state limit  $G(X_i)$ , variables of design  $X_i$  chosen as random variables. The considered variables of design are the viscosity, the angular speed, the length and diameter of the bearing, the radial clearance. The domains of the performance function [8, 9] are defined as:

$G(X_i) > 0$  is the domain of safety;

$G(X_i) < 0$  is the domain of failure;

$G(X_i) = 0$  is the limit state;

Hasofer and Lind [12] show that the index of reliability is the minimum of the distance between the origin and the space of variables normalized with the constraint  $H(U_i)$  where  $H$  the function of performance in the reduced centered standardized space (see Fig. 4). The calculation of the index of reliability requires the research for the most likely point of failure  $P^*$  called design point when it is considered as reference point for a sizing. The evaluation of this point of failure is a matter of a not linear optimization adapted to the nature of the problem.

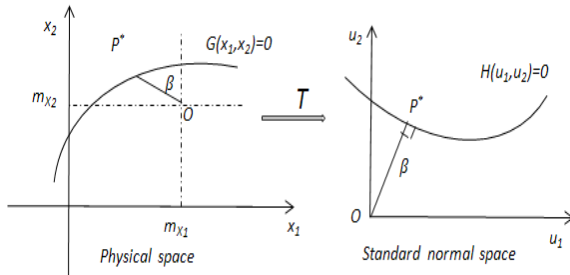


Fig. 4. Transformation into standard normal space

$m_{x1}$  and  $m_{x2}$  are respectively the average of variables  $X_1$  and  $X_2$  in the physical space.  $T$  is the transformation of the passage of the physical space in the standardized space.

In the case of our study, we are going to consider that the random variables are Gaussian and independents. The transformation  $T$  of the physical space is immediate and builds itself variable by variable.

$$U_i = T_i(X_i)$$

$$X_i \xrightarrow{T} U_i = \frac{x_i - m_{x_i}}{\sigma_{x_i}} \quad (23)$$

$X_i$ : Random variable in the physical space

$U_i$ : Random variable in the gaussian standardized space

$m_{x_i}$ : Mean of the random variable  $X_i$

$\sigma_{x_i}$ : Standard deviation of the random variable  $X_i$

$\beta$  Index of reliability estimated in the centered standardized space.

- Algorithm of Rackwitz-Fiessler

The algorithm developed by Rackwitz and Fiessler is based on the calculation of the gradients of every variable. Knowing the gradients, we can then estimate linearly the design point, and take the same strategy around this new point  $P^*$  until convergence.

By taking place in the point  $U^k$  corresponding to the design point of the iteration  $k$ , we can write the development of Taylor around this point of the function  $H(U)$  [8]:

$$H(U) = H(U^{*k}) + \nabla H(U)^T_{U^{*k}} (U - U^{*k}) \quad (24)$$

By this formula, we can estimate the design of the following iteration:

$$H(U^{*k+1}) = H(U^{*k}) + \nabla H(U)^T_{U^{*k}} (U^{*k+1} - U^{*k}) = 0 \quad (25)$$

We introduce the vector of the cosine director  $\alpha$ :

$$\alpha = \frac{\nabla H(U)}{\|\nabla H(U)\|} \quad (26)$$

The limit state takes the following shape:

$$\frac{\nabla H(U^{*k})}{\|\nabla H(U)\|_{U^{*k}}} + (U^{*k+1} - U^{*k})^T \alpha^k = 0 \quad (27)$$

Or

$$(U^{*k+1})^T \alpha^k = (U^{*k})^T \alpha^k - \frac{\nabla H(U^{*k})}{\|\nabla H(U)\|_{U^{*k}}} \quad (28)$$

By introducing the index of reliability into the last equation, it is transformed by:

$$\beta^{k+1} = (U^{*k})^T \alpha^k - \frac{\nabla H(U^{*k})}{\|\nabla H(U)\|_{U^{*k}}} \quad (29)$$

We can then estimate the new design point:

$$U^{*k+1} = -\beta^{k+1} \alpha^k \quad (30)$$

The algorithm stops when:

$$\|\beta^{k+1} - \beta^k\| < \varepsilon_s \quad (31)$$

Where  $\varepsilon_s$  is the desired condition of stop. In the cases of the mechanical calculations, the evaluation of the gradients by finished differences appeals to a numeral calculation. We define a point to be calculated for every variable:

$$U_j^k = (U_1^{*k}, U_2^{*k}, \dots, U_n^{*k})$$

Variable  $U_j^k$  written in the standardized space is transformed at first into variables  $X_j^k$  in the physical space. Then, we make the numeral calculation associated in every point  $U_j^k$  to estimated  $H(U_j^k)$ . The calculation of the function of performance  $H$  for the design point  $U^{*k}$  is also led. The gradient  $H(U^{*k})$  is then given as:

$$\nabla H(U^{*k})_j = \frac{H(U_j^{*k}) - H(U^{*k})}{\|U_j^{*k} - U^{*k}\|} \quad (32)$$

The use of this plan implies  $n + 1$  calculations by iteration of the algorithm.

- FORM

FORM (First Order Reliability Method) approximates the domain of failure by a half-space bounded by hyper one tangent plan on the surface in the design point. Of the fact of symmetry of revolution of the standardized normal multi-distribution, the probability of failure is simply approached by:

$$P_f = \varphi(-\beta) \quad (33)$$

The design point is determined by looking for the point of limit state the closest to the origin of the standardized space. The design point is the solution of the problem of optimization:

$$\begin{cases} \beta = \min(\sqrt{U^t U}) \\ H(U) = 0 \end{cases} \quad (34)$$

The result of this problem of minimization under constraint will be solved by the algorithm of Rackwitz-Fiessler and the design point estimated as:

$$U^* = -\alpha^t \beta \quad (35)$$

The normalized gradient  $\alpha$  to the function at the limit state, estimated at the point of design  $U^*$  is determined by (26): The index of reliability  $\beta$  is determined by (29): The equation of the tangent hyperplan in the design point  $U^*$  is:

$$\tilde{H}(U) = \beta + \sum_{i=1}^n \alpha_i U_i \quad (36)$$

This method supplies an exact result when the state-limit is linear in the standard space. it becomes indistinct when the function of performance is strongly not linear in the neighborhood of the point of design point or when there are secondary significant minimums.

- SORM

The SORM (Second Order Reliability Method) consists in approaching the surface of state-limit by a quadratic surface. For that purpose, we make a Taylor development of the

performance function in order two at the design point  $U^*$ . This method consists to determine an approximation of the function performance  $H(U)$ , noted  $\tilde{H}(U)$  by a development of Taylor around a given point  $U_0$ .

$$\tilde{H}(U) = H(U_0) + a^t(U - U_0) + \frac{1}{2}(U - U_0)^t \mathbb{H}(U_0)(U - U_0) + O(\|U - U_0\|^2) \quad (37)$$

The matrix Hessian  $\mathbb{H}$  owes to be determined then diagonalized so that the main curvatures  $k_i$  can be calculated. The approximation of these curvatures allows having a quadratic approximation SORM which thus takes the shape of hyper tangent paraboloid in the design point and which can be expressed:

$$\tilde{H}(U) = U_n - \beta - \frac{1}{2} \sum_{i=1}^{n-1} k_i U_i^2 \quad (38)$$

The probability of failure can be so estimated by the following relation:

$$P_f = \varphi(-\beta) \prod_{i=1}^{n-1} (1 + k_i \beta)^{-\frac{1}{2}} \quad (39)$$

Surrounding areas of second order were envisaged by making the hypothesis that a development of the state limit in the second order was better than a development in the first order.

- Monte Carlo method

Methods by simulation allow estimating the probability of failure in the case of complex laws of probability, correlations between variables or function of not linear limit states. However these methods require calculation time which can be prohibitive. The principle of the simulations of Monte Carlo is to do, according to the law of joint probability of the random vector and to count the number of times when the system is in the domain of failure. The probability of failure can be expressed by the relation:

$$P_f \approx \frac{1}{N} \sum_{i=1}^N I[G(X_i) \leq 0] \quad (40)$$

Where  $X_i$  the vector of random variables, and the indicator function  $I$  is equal 1 if the condition  $G(X_i) \leq 0$  is true and 0 if not. The evaluation of the probability of failure is exact if the number of samples is sufficiently high. One of the major inconveniences of the methods of Monte Carlo is the large number of simulations required in certain cases. Indeed, for a low probability of failure, an inadequate number of simulations could lead to a significant degree of error.

## VIII. APPLICATIONS

We consider five random variables as shown in table 1.

Table.1. Random variables

| Variables<br>$X_i$   | Mean<br>$m_{X_i}$ | Standard<br>deviation<br>$\sigma_{X_i}$ | Distribution |
|----------------------|-------------------|---|--------------|
| $\mu(Pa.s)$          | $12E-4$           | $12E-5$                                 | Normal       |
| $\omega(radian.s-1)$ | 157               | 15.7                                    | Normal       |
| $L(m)$               | 0.5               | $1E-5$                                  | Normal       |
| $R(m)$               | $5E-2$            | $1E-4$                                  | Normal       |
| $C(m)$               | $40E-6$           | $40E-7$                                 | Normal       |

For long journal bearing with conditions of Sommerfeld, the function of performance  $G$  is defined as a difference between a critical load capacity and an operating load capacity [1, 2, 3 and 4].

$$G(L, R, C, \omega, \mu) = W_S^c - W_S$$

The critical load capacity with conditions of Sommerfeld is taken for a value of relative eccentricity:

$$\varepsilon = 0.95$$

$$W_S^c = 2.9E5 \text{ N.}$$

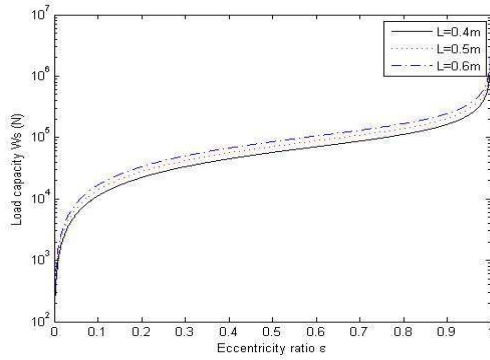


Fig. 5 Load capacity versus relative eccentricity with Sommerfeld conditions

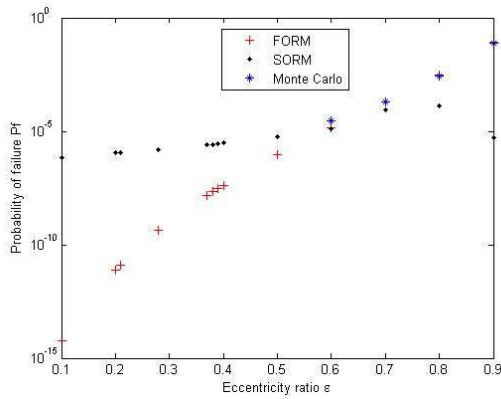


Fig. 6 Failure probability according versus eccentricity with conditions of Sommerfeld

For the long journal bearing on conditions of Gumbel, the function of performance is the difference between a critical load capacity and an operating load capacity.

$$\varepsilon = 0.95$$

$$W_G^c = 2.1E5 \text{ N}$$

It writes then:

$$G(L, R, C, \omega, \mu) = W_G^c - W_G$$

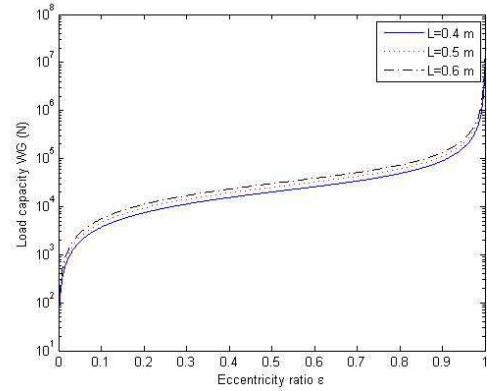


Fig. 7 Load capacity versus relative eccentricity with Gumbel conditions

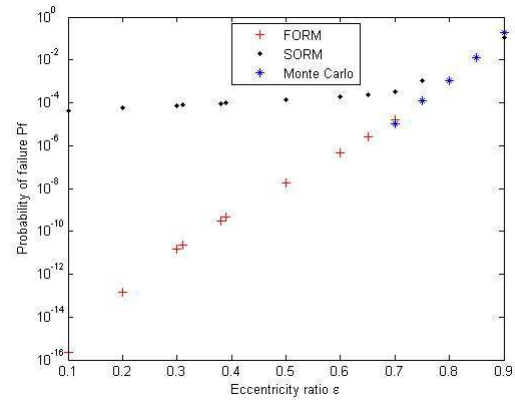


Fig. 8 Failure probability according versus eccentricity with conditions of Gumbel

For the long journal bearing on conditions of Reynolds, the function of performance is also given as the difference between the critical load capacity and an operating load capacity.

$$G(L, R, C, \omega, \mu) = W_R^c - W_R$$

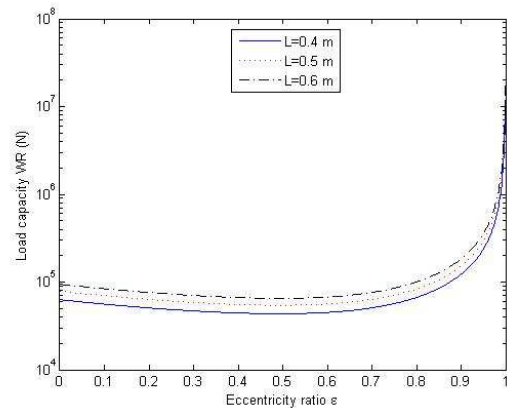


Fig.9 Load capacity versus relative eccentricity with Reynolds conditions



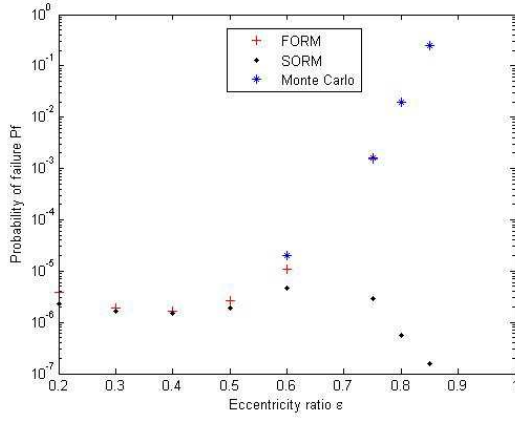


Fig.10 Failure probability according versus eccentricity with conditions of Reynolds

For the fluid break, we choose  $\psi_R = 4.21346$  rad.

The load capacity becomes rather important with the increase of the length of the journal bearing that is the ratio of the length for the diameter upper to 4 ( $L/D > 4$ ). For the approximation

of Sommerfeld, the probability of failure is estimated according to a critical load capacity  $W_s^C = 2.9E5$  N corresponding in a relative eccentricity  $\varepsilon = 0.95$ . The probability of failure is lower than  $10^{-3}$  for lower eccentricities than 0.6 according to the results given by FORM and SORM. The results of Monte Carlo for the higher eccentricities than 0.6 give values of probability of failure in agreement with FORM results. However, the results of three methods (FORM, SORM and Monte Carlo) are almost similar for eccentricities higher than 0.6. The probability of failure increases for a decrease of the index of reliability which represents the distance of the origin to the point design. The values of index of reliability are acceptable only for eccentricities higher than 0.6. Otherwise the result would not be in agreement with the probability of failure calculated with the three methods.

For the approximation of Gumbel, the probability of failure remains acceptable for values of eccentricities higher than 0.7. Beyond these values, the probability of failure for the three methods are in agreement. The probability of failure increases with the relative eccentricity and decreases in an exponential way with the index of reliability.

For the approximation of Reynolds, the probability of failure remains practically constant for lower eccentricities than 0.5. The probability of failure decreases exponentially with the increase of the values of index of reliability in the case of the three methods (FORM, SORM and Monte Carlo). The results of the probability of failure according to the index reliability for these three methods are perfectly similar.

In the case of short journal bearings, the best adapted conditions are the ones of Gumbel according to Dubois and al. [13]. This hypothesis of short journal bearing justifies itself for a ratio of the length in the lower diameter 1/8. The function of performance is given by:

$$G(L, R, C, \omega, \mu) = W_C^e - W_C$$

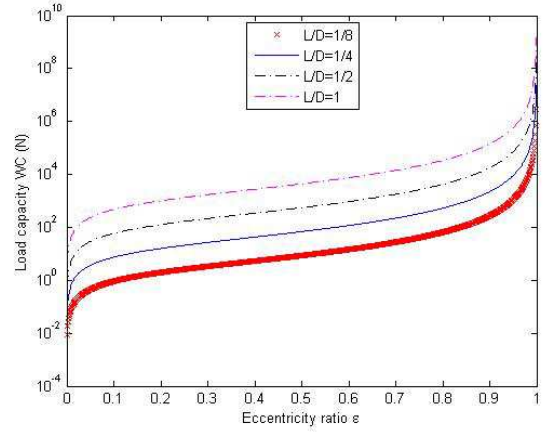


Fig. Load capacity versus relative eccentricity with conditions of short bearing

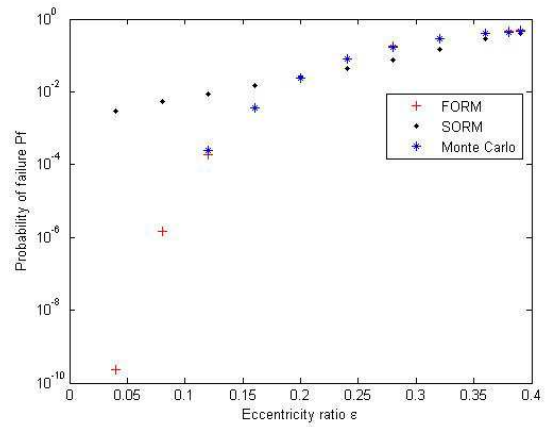


Fig.12 10 Failure probability according versus eccentricity with conditions of short bearing

For the short journal bearing, the load capacity increases slightly with the eccentricity. The results obtained from the probability of failure are estimated according to the hypothesis of a short bearing. The significant values of the probability of failure are obtained for eccentricities higher than 0.15 corresponding to index of reliability lower than 3. However, for each method, the evolution of the probability of failure according to the eccentricity are nearby beyond  $\varepsilon = 0.25$  corresponding to a probability of failure higher than  $10^{-2}$ .

## Conclusion

We proposed in this paper a suitable methodology to estimate the reliability of a journal bearing. An analytical approach for the calculation of load capacity of a journal bearing is developed with a combination of the principle of reliability. FORM, SORM and Monte Carlo simulation are used to estimate the failure probability of a journal bearing.

Among these three methods, only FORM and Monte Carlo are more nearby to compare with SORM. As FORM and SORM are approximations, the calculation of the derivative becomes difficult and particularly with SORM. The used method of Monte Carlo is the important sampling. This technique of



simulation consists in making drawings for the neighborhood of the design point where the density of probability is more important. These studies of reliability of the cylindrical bearing allow us to make a decision from the point of view of the design point and a better choice for the customer.

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